Theorem 1.1 Some Basic Limits

Let b and c be real numbers, and let n be a positive integer.

1. $\lim_{x \to c} b = b$
2. $\lim_{x \to c} x = c$
3. $\lim_{x \to c} x^n = c^n$

Theorem 1.2 Properties of Limits

Let b and c be real numbers, let n be a positive integer, and let f and g be functions with the limits $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = K$.

1. Scalar multiple:	$\lim_{x \to c} [bf(x)] = bL$
2. Sum or difference:	$\lim_{x \to c} [f(x) \pm g(x)] = L \pm K$
3. Product:	$\lim_{x \to c} [f(x)g(x)] = LK$
4. Quotient:	$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{K}, \quad K \neq 0$
5. Power:	$\lim_{x \to c} [f(x)]^n = L^n$

The proof of Property 1 of this theorem is left as an exercise. (See Exercise 115.) The proofs of the other four properties are given in Appendix A.

Theorem 1.3 Limits of Polynomial and Rational Functions

If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c).$$

If r is a rational function given by $r(x) = p(x)/q(x)$ and c is a real number such that $q(c) \neq 0$, then

$$\lim_{x \to c} r(x) = r(c) = \frac{p(c)}{q(c)}.$$

Theorem 1.4 The Limit of a Function Involving a Radical

Let n be a positive integer. The limit below is valid for all c when n is odd, and is valid for $c > 0$ when n is even.

$$\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$$

A proof of this theorem is given in Appendix A.

Theorem 1.5 The Limit of a Composite Function

If f and g are functions such that

$$\lim_{x \to c} g(x) = L$$

and

$$\lim_{x \to L} f(x) = f(L)$$

then

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L).$$

A proof of this theorem is given in Appendix A.

Theorem 1.6 **Limits of Transcendental Functions**

Let c be a real number in the domain of the given transcendental function.

1. $\lim_{x \to c} \sin x = \sin c$
2. $\lim_{x \to c} \cos x = \cos c$
3. $\lim_{x \to c} \tan x = \tan c$
4. $\lim_{x \to c} \cot x = \cot c$
5. $\lim_{x \to c} \sec x = \sec c$
6. $\lim_{x \to c} \csc x = \csc c$
7. $\lim_{x \to c} a^x = a^c, a > 0$
8. $\lim_{x \to c} \ln x = \ln c$

Theorem 1.7 **Functions That Agree at All but One Point**

Let c be a real number, and let $f(x) = g(x)$ for all $x \neq c$ in an open interval containing c. If the limit of $g(x)$ as x approaches c exists, then the limit of $f(x)$ also exists and

$$\lim_{x \to c} f(x) = \lim_{x \to c} g(x).$$

A proof of this theorem is given in Appendix A.

Theorem 1.8 The Squeeze Theorem

If $h(x) \leq f(x) \leq g(x)$ for all x in an open interval containing c, except possibly at c itself, and if

$$\lim_{x \to c} h(x) = L = \lim_{x \to c} g(x)$$

then $\lim_{x \to c} f(x)$ exists and is equal to L.

A proof of this theorem is given in Appendix A.

Theorem 1.9 Three Special Limits

1. $\lim_{x \to 0} \frac{\sin x}{x} = 1$
2. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
3. $\lim_{x \to 0} (1 + x)^{1/x} = e$

Remark

The third limit of Theorem 1.9 will be used in Chapter 2 in the development of the formula for the derivative of the exponential function $f(x) = e^x$.