2.6 Solving Absolute Value Inequalities

GO DIGITAL

Learning Target Write and solve inequalities involving absolute value.

Success Criteria

- I can write a compound inequality related to a given absolute value inequality.
- I can solve absolute value inequalities.
- I can use absolute value inequalities to solve real-life problems.

EXPLORE IT! Solving an Absolute Value Inequality

Work with a partner. Consider the absolute value inequality

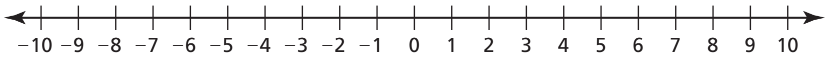
$$|x + 2| \le 3.$$

a. Explain what you think this inequality means.

b. Can you find a number that makes the inequality true? If so, what is the number?

c. Do you think there are other numbers that make the inequality true? If so, find several of them. Compare your answers with your classmates.

d. On the real number line below, locate the point for which the expression $|x + 2|$ is equal to 0.



Then locate the numbers you found in parts (b) and (c) on the real number line. What do you notice?

e. Can you write two linear inequalities that use the expression $x + 2$ to represent the solutions of $|x + 2| \le 3$? Explain.

f. Repeat parts (b)–(e) for the inequality $|x + 2| \ge 3$. Compare your results with those for the inequality $|x + 2| \le 3$.

g. Describe how to find the solutions of the absolute value inequalities algebraically. Then find the solutions.

i. $|x - 4| \le 2$

ii. $|x - 4| \ge 2$

h. **CHOOSE TOOLS** Solve the absolute value inequalities in part (g) in a different way. Explain your method.

x_1	$\lvert x_1 - 4\rvert$
−6	10
−5	9
−4	
−3	
−2	
−1	
0	
1	
2	

Maintain Oversight
How can you change one of the absolute value inequalities shown so that it has no solution?

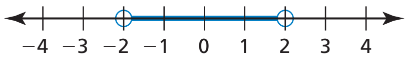
Solving Absolute Value Inequalities

An **absolute value inequality** is an inequality that contains an absolute value expression. For example, $|x| < 2$ and $|x| > 2$ are absolute value inequalities. Recall that $|x| = 2$ means the distance between x and 0 is 2.

Vocabulary

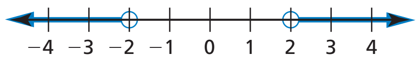
absolute value inequality, *p. 108*
absolute deviation, *p. 110*

The inequality $|x| < 2$ means the distance between x and 0 is *less than* 2.



The graph of $|x| < 2$ is the graph of $x > -2$ *and* $x < 2$.

The inequality $|x| > 2$ means the distance between x and 0 is *greater than* 2.



The graph of $|x| > 2$ is the graph of $x < -2$ *or* $x > 2$.

You can solve these types of inequalities by solving a compound inequality.

KEY IDEA

Solving Absolute Value Inequalities

Let c be a positive real number.

To solve $|ax + b| < c$, solve the compound inequality

$ax + b > -c$ *and* $ax + b < c$.

To solve $|ax + b| > c$, solve the compound inequality

$ax + b < -c$ *or* $ax + b > c$.

In the inequalities above, you can replace $<$ with $\leq$ and $>$ with $\geq$.

EXAMPLE 1 Solving Absolute Value Inequalities

WATCH

Solve each inequality. Graph each solution, if possible.

a. $|x + 7| \leq 2$ **b.** $|8x - 11| < 0$

SOLUTION

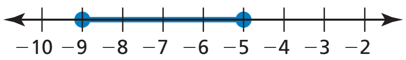
a. Use $|x + 7| \leq 2$ to write a compound inequality. Then solve.

$x + 7 \geq -2$	*and*	$x + 7 \leq 2$	Write a compound inequality.
$\underline{-7} \quad \underline{-7}$		$\underline{-7} \quad \underline{-7}$	Subtraction Property of Inequality
$x \geq -9$	*and*	$x \leq -5$	Simplify.

REMEMBER

A compound inequality with "and" can be written as a single inequality.

▶ The solution is $-9 \leq x \leq -5$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|8x - 11|$ cannot be less than 0.

▶ So, the inequality has no solution.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. WRITING How do you determine whether to use a compound inequality with "and" or a compound inequality with "or" when solving an absolute value inequality?

Solve the inequality. Graph the solution, if possible.

2. $|x| \leq 3.5$ **3.** $|k - 3| < -1$ **4.** $\left|\frac{1}{2}w - 1\right| < 11$

EXAMPLE 2 Solving Absolute Value Inequalities

WATCH

Solve each inequality. Graph each solution.

a. $|c - 1| \geq 5$ **b.** $|10 - m| \geq -2$ **c.** $4|2x - 3| + 1 > 17$

SOLUTION

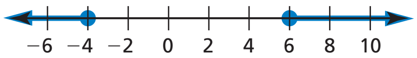
a. Use $|c - 1| \geq 5$ to write a compound inequality. Then solve.

$$c - 1 \leq -5 \quad or \quad c - 1 \geq 5 \qquad \text{Write a compound inequality.}$$

$$\underline{+1} \quad \underline{+1} \qquad \underline{+1} \quad \underline{+1} \qquad \text{Addition Property of Inequality}$$

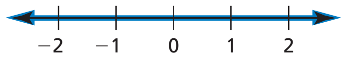
$$c \leq -4 \quad or \quad c \geq 6 \qquad \text{Simplify.}$$

▶ The solution is $c \leq -4$ *or* $c \geq 6$.



b. By definition, the absolute value of an expression must be greater than or equal to 0. The expression $|10 - m|$ will always be greater than -2.

▶ So, all real numbers are solutions.



c. First isolate the absolute value expression on one side of the inequality.

$$4|2x - 3| + 1 > 17 \qquad \text{Write the inequality.}$$

$$\underline{-1} \quad \underline{-1} \qquad \text{Subtraction Property of Inequality}$$

$$4|2x - 3| > 16 \qquad \text{Simplify.}$$

$$\frac{4|2x - 3|}{4} > \frac{16}{4} \qquad \text{Division Property of Inequality}$$

$$|2x - 3| > 4 \qquad \text{Simplify.}$$

Use $|2x - 3| > 4$ to write a compound inequality. Then solve.

$$2x - 3 < -4 \quad or \quad 2x - 3 > 4 \qquad \text{Write a compound inequality.}$$

$$\underline{+3} \quad \underline{+3} \qquad \underline{+3} \quad \underline{+3} \qquad \text{Addition Property of Inequality}$$

$$2x < -1 \qquad 2x > 7 \qquad \text{Simplify.}$$

$$\frac{2x}{2} < \frac{-1}{2} \qquad \frac{2x}{2} > \frac{7}{2} \qquad \text{Division Property of Inequality}$$

$$x < -\frac{1}{2} \quad or \quad x > \frac{7}{2} \qquad \text{Simplify.}$$

▶ The solution is $x < -\frac{1}{2}$ *or* $x > \frac{7}{2}$.

SELF-ASSESSMENT

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Solve the inequality. Graph the solution.

5. $|x + 3| > 8$ **6.** $|n + 2| - 3.7 \geq -6$ **7.** $3|d + 1| - 7 \geq -2$

8. WRITING Describe how solving $|w - 9| \leq 2$ is different from solving $|w - 9| = 2$.

9. REASONING What is the solution of the inequality $|ax + b| < c$, where $c < 0$? What is the solution of the inequality $|ax + b| > c$, where $c < 0$? Explain.

Solving Real-Life Problems

WORDS AND MATH

A *deviation* is something that is different from the expected norm. In mathematics, *absolute deviation* represents how far a number deviates from a specific value.

The **absolute deviation** of a number x from a given value is the absolute value of the difference of x and the given value.

$$\text{absolute deviation} = |x - \text{given value}|$$

EXAMPLE 3 **Modeling Real Life** WATCH INFO

A mountain climber wants to buy a new camera drone to help map out a safe route to a mountain's summit. The table shows the prices of several camera drones. The climber is willing to pay the mean price with an absolute deviation of at most \$100. How many of the camera drone prices meet this condition?

Camera Drone Prices	
\$890	\$750
\$650	\$370
\$660	\$670
\$450	\$650
\$725	\$825

SOLUTION

1. **Understand the Problem** You know the prices of 10 camera drones. You are asked to find how many drones are at most \$100 from the mean price.
2. **Make a Plan** Calculate the mean price by dividing the sum of the prices by the number of prices. Use the absolute deviation and the mean price to write an absolute value inequality. Then solve the inequality and use it to answer the question.
3. **Solve and Check**

The mean price is $\frac{6640}{10} = \$664$. Let x represent a price the climber is willing to pay.

$|x - 664| \le 100$ Write the absolute value inequality.

$-100 \le x - 664 \le 100$ Write a compound inequality.

$564 \le x \le 764$ Add 664 to each expression and simplify.

▶ The prices the climber is willing to pay are at least \$564 and at most \$764. Six prices meet this condition: \$750, \$650, \$660, \$670, \$650, and \$725.

STUDY TIP

The absolute deviation of at most \$100 from the mean, \$664, is given by the inequality $|x - 664| \le 100$.

Check Reasonableness You can check that your answer is correct by graphing the drone prices and the mean on a number line. Any point within 100 of 664 represents a price that the climber is willing to pay.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. **WHAT IF?** The climber is willing to pay the mean price with an absolute deviation of at most \$75. Do you expect the number of prices that meet this condition to increase or decrease? Explain your reasoning. How many of the camera drone prices meet this condition?

11. A softball team is participating in a tournament where the team will spend three nights at a hotel. Each hotel offers a 50% discount for the third night. The coach wants to keep the total cost for each player at \$225 with an absolute deviation of at most \$25. Write and solve an absolute value inequality to find which hotels meet this condition.

Hotel	Price per night
Hotel A	\$80
Hotel B	\$105
Hotel C	\$75
Hotel D	\$90