

Ex $f(x) = |3x + 2| + 1$

x-value of vertex

$$0 = 3x + 2$$

$$-2 = 3x$$

$$-\frac{2}{3} = x$$

y-value of vertex

$$y = |3(-\frac{2}{3}) + 2| + 1$$

$$y = 0 + 1$$

$$y = 1$$

vertex @ $(-\frac{2}{3}, 1)$

3 is HT \rightarrow

2 is HT \leftarrow

1 is VT \uparrow

Domain $\rightarrow \{x \mid -\infty < x < +\infty\}$
 Range $\rightarrow \{y \mid 1 \leq y < +\infty\}$

3.8 ¹⁵ Practicewith
Calc Chat and Calc View

GO DIGITAL

In Exercises 1–8, graph the function. Compare the graph to the graph of $f(x) = |x|$. Find the domain and range. ▶ Examples 1 and 2

1. $d(x) = |x| - 4$

2. $r(x) = |x| + 5$

3. $m(x) = |x + 1|$

4. $v(x) = |x - 3|$

5. $p(x) = \frac{1}{3}|x|$

6. $j(x) = 3|x|$

7. $a(x) = -5|x|$

8. $q(x) = -\frac{3}{2}|x|$

In Exercises 9–12, write an equation that represents the given transformation(s) of the graph of $g(x) = |x|$.

9. vertical translation 7 units down

10. horizontal translation 10 units left

11. vertical shrink by a factor of $\frac{1}{4}$ 12. vertical stretch by a factor of 3 and a reflection in the x -axis

$$f(x) = |x| - 7$$

$$f(x) = |x + 10|$$

$$f(x) = \frac{1}{4}|x|$$

$$f(x) = -3|x|$$

In Exercises 13–18, graph and compare the two functions. ▶ Example 3

13. $f(x) = |x - 4|$; $g(x) = |3x - 4|$

14. $h(x) = |x + 5|$; $t(x) = |2x + 5|$

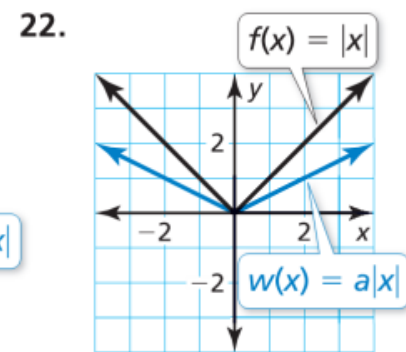
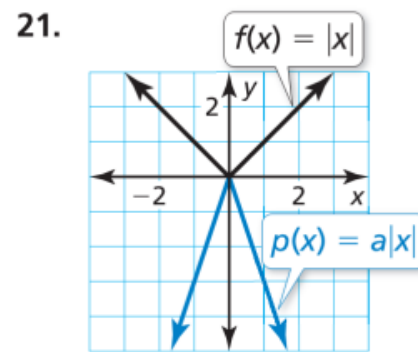
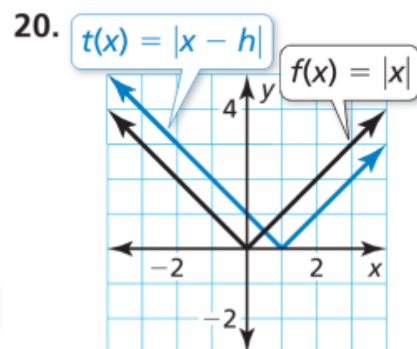
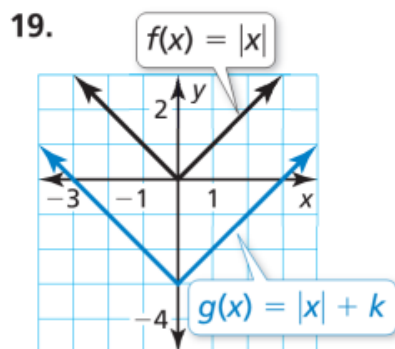
15. $p(x) = |x + 1| - 2$; $q(x) = \left|\frac{1}{4}x + 1\right| - 2$

16. $w(x) = |x - 3| + 4$; $y(x) = |5x - 3| + 4$

17. $a(x) = |x + 2| + 3$; $b(x) = |-4x + 2| + 3$

18. $u(x) = |x - 1| + 2$; $v(x) = \left|-\frac{1}{2}x - 1\right| + 2$

In Exercises 19–22, compare the graphs. Find the value of h , k , or a .



In Exercises 23–30, graph the function. Then describe the transformations from the graph of $f(x) = |x|$ to the graph of the function. ▶ Example 4

23. $r(x) = |x + 2| - 6$

24. $c(x) = |x + 4| + 4$

25. $d(x) = -|x - 3| + 5$

26. $v(x) = -3|x + 1| + 4$

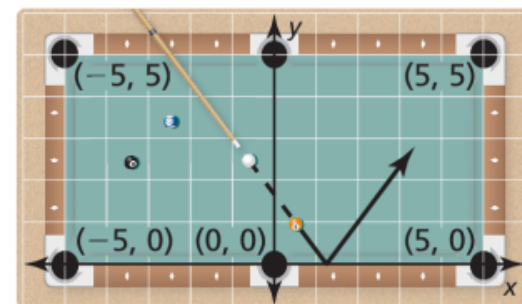
27. $m(x) = \frac{1}{2}|x + 4| - 1$

28. $s(x) = |2x - 2| - 3$

29. $j(x) = |-x + 1| - 5$

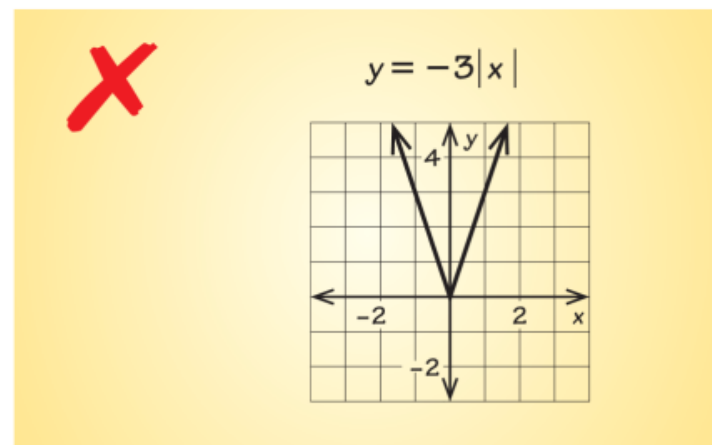
30. $n(x) = \left|-\frac{1}{3}x + 1\right| + 2$

31. **MODELING REAL LIFE** On the pool table shown, you bank the five ball off the side represented by the x -axis. The path of the ball is modeled by the function $p(x) = \frac{4}{3}\left|x - \frac{5}{4}\right|$.



- a. At what point does the five ball bank off the side?
b. Do you make the shot? Explain your reasoning.

32. **ERROR ANALYSIS** Describe and correct the error in graphing the function.



$$8) \quad g(x) = -\frac{3}{2}|x|$$

X-Value Vertex

$$x = 0$$

Y-Value Vertex

$$y = 0$$

Vertex @ (0,0)

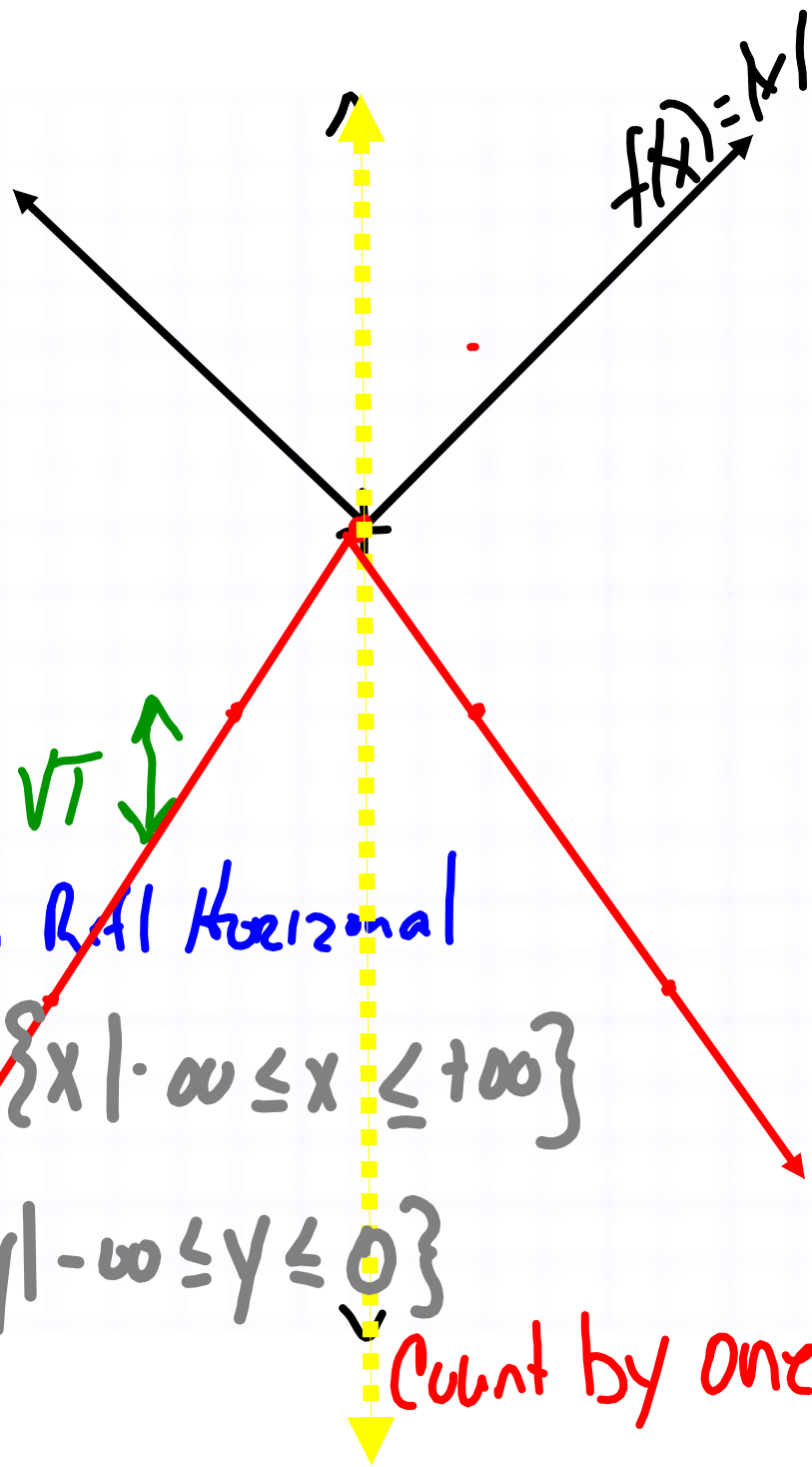
$\frac{3}{2}$ is $\sqrt{1}$

- is Real Horizontal

$$\text{Domain} \Rightarrow \{x | -\infty \leq x \leq +\infty\}$$

$$\text{Range} \Rightarrow \{y | -\infty \leq y \leq 0\}$$

Count by ones



$$5) p(x) = \frac{1}{3} |x|$$

x-value vertex x

$$x = 0$$

y-value vertex

$$y = 0$$

Vertex (0, 0)

$$p(2) = \frac{1}{3} |2|$$

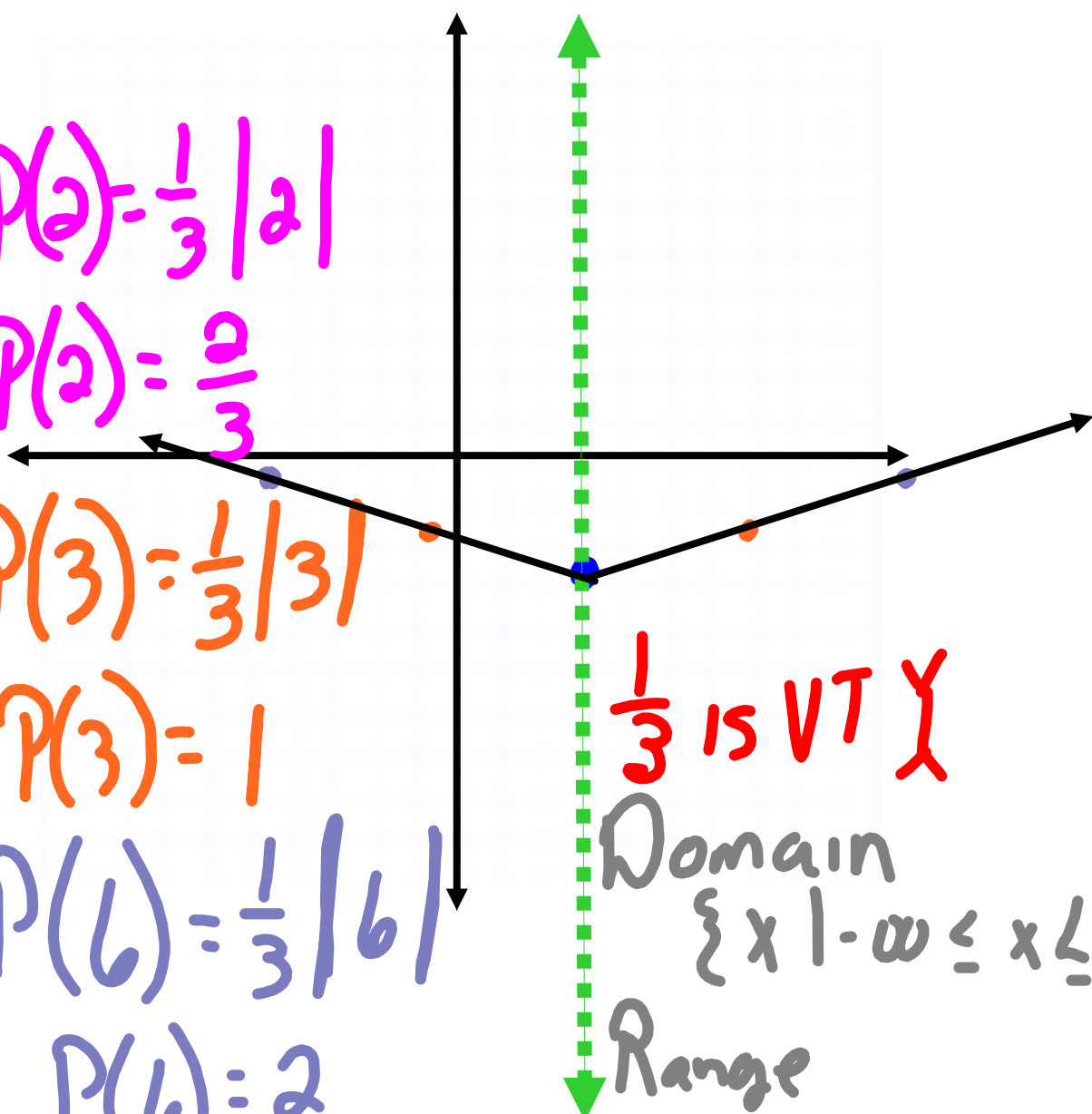
$$p(2) = \frac{2}{3}$$

$$p(3) = \frac{1}{3} |3|$$

$$p(3) = 1$$

$$p(6) = \frac{1}{3} |6|$$

$$p(6) = 2$$



$\frac{1}{3}$ is VTY

Domain

$$\{x \mid -\infty \leq x \leq +\infty\}$$

Range

$$\{y \mid 0 \leq y \leq +\infty\}$$

13. $f(x) = |x - 4|$; $g(x) = |3x - 4|$

X-value Vertex

$$0 = x - 4$$

$$4 = x$$

Y-value Vertex

$$y = 0$$

Vertex (4, 0)

$$f(2) = |2 - 4| = 2 \quad (2, 2)$$

$$f(3) = |3 - 4| = 1 \quad (3, 1)$$

X-value Vertex

$$0 = 3x - 4$$

$$\frac{4}{3} = x$$

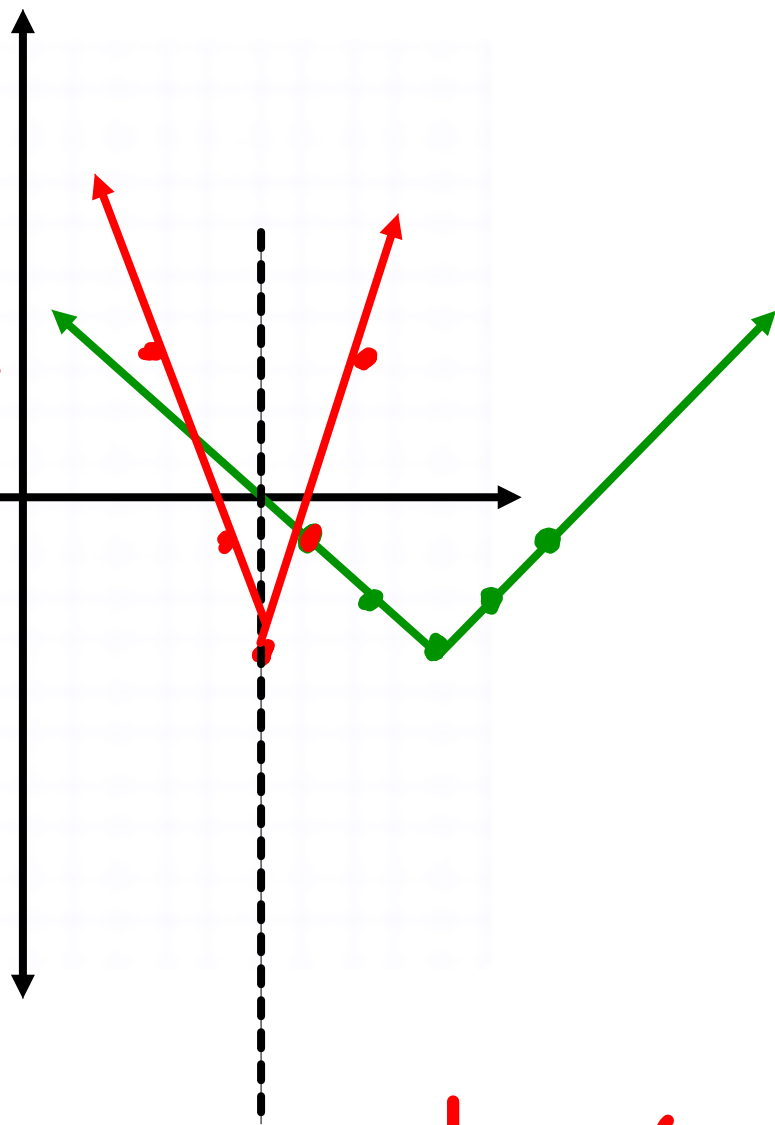
Y-value Vertex

$$y = 0$$

Vertex $(\frac{4}{3}, 0)$

$$g(2) = |3(2) - 4| = 2 \quad (2, 2)$$

$$g(3) = |3(3) - 4| = 5 \quad (3, 5)$$



- 36. REASONING** An absolute value function is positive over its entire domain. How many x -intercepts does the graph of the function have?
- 37. MODELING REAL LIFE** A traveler is driving from Nevada to Arizona. The function $d(t) = 60|t - 2.5|$ represents the distance (in miles) the car is from the state line after t hours.
- Graph the function and interpret the intercepts.
 - When is the function decreasing? increasing? Explain what each represents in this context.
 - A rest stop is 90 miles from the state line. How long will it take the traveler to reach the rest stop? Explain.

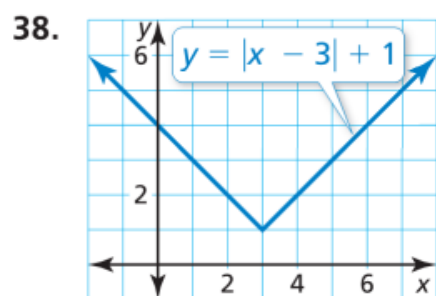
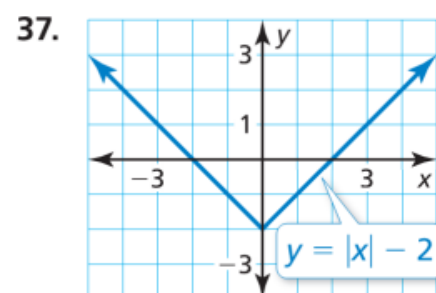
- 38. STRUCTURE** The points $A(-\frac{1}{2}, 3)$, $B(1, 0)$, and $C(-4, -2)$ lie on the graph of the absolute value function f . Find the coordinates of the points corresponding to A , B , and C on the graph of each function.

- $g(x) = f(x) - 5$
- $h(x) = f(x - 3)$
- $j(x) = -f(x)$
- $k(x) = 4f(x)$

- 39. STRUCTURE** Explain how the graph of each function compares to the graph of $y = |x|$ for positive and negative values of k , h , and a .

- $y = |x| + k$
- $y = |x - h|$
- $y = a|x|$
- $y = |ax|$

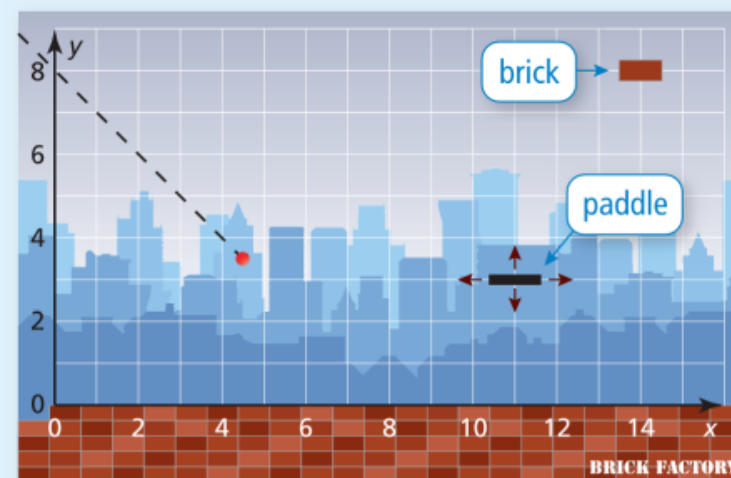
CONNECTING CONCEPTS In Exercises 37 and 38, write an absolute value function whose graph intersects the given graph to form an enclosed square.



- 40. REASONING** Is it possible for an absolute value function to always be increasing? decreasing? Explain your reasoning.

40. HOW DO YOU SEE IT?

The object of a computer game is to break bricks by deflecting a ball toward them using a paddle. The graph shows the current path of the ball and the location of the last brick.



- You can move the paddle up, down, left, and right. At what coordinates should you place the paddle to break the last brick? Assume the ball deflects at a right angle.
- You move the paddle to the coordinates in part (a), and the ball is deflected. How can you write an absolute value function that describes the path of the ball?

In Exercises 41–44, graph the function. Then rewrite the absolute value function as two linear functions, one that has the domain $x < 0$ and one that has the domain $x \geq 0$.

- $y = |x|$
- $y = |x| - 3$
- $y = -|x| + 9$
- $y = -4|x|$

In Exercises 45–48, sketch a graph of an absolute value function that has the given characteristics. Then write the function represented by your graph.

- vertex $(3, -4)$, x -intercepts: 1 and 5
- vertex $(-\frac{3}{2}, 3)$, x -intercepts: $-\frac{5}{2}$ and $-\frac{1}{2}$
- vertex $(-6, -1)$, y -intercept: -5
- x -intercept: -2 , y -intercept: $\frac{1}{2}$

- 49. COLLEGE PREP** Which of the following functions have the same end behavior?

- $a(x) = 4|x - 2| + 3$
- $b(x) = -\frac{3}{2}|x + 5|$
- $c(x) = |-2x - 6| - 1$
- $d(x) = |9 - \frac{1}{2}x|$

✖. THOUGHT PROVOKING

Graph an absolute value function f that represents the route a wide receiver runs in a football game. Let the x -axis represent distance (in yards) across the field horizontally. Let the y -axis represent distance (in yards) down the field. Limit the domain so the route is realistic.

In Exercises 51–54, graph and compare the two functions.

51. $f(x) = |x - 1| + 2$; $g(x) = 4|x - 1| + 8$
 52. $s(x) = |2x - 5| - 6$; $t(x) = \frac{1}{2}|2x - 5| - 3$
 53. $v(x) = -2|3x + 1| + 4$; $w(x) = 3|3x + 1| - 6$
 54. $c(x) = 4|x + 3| - 1$; $d(x) = -\frac{4}{3}|x + 3| + \frac{1}{3}$

- ✖. **USING TOOLS** Graph $y = 2|x + 2| - 6$ and $y = -2$ in the same coordinate plane. Use the graph to solve the equation $2|x + 2| - 6 = -2$. Use technology to check your solutions.

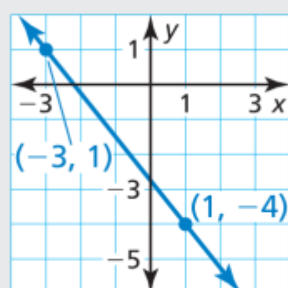
- ✖. **REASONING** Describe the transformations from the graph of $g(x) = -2|x + 1| + 4$ to the graph of $h(x) = |x|$. Explain your reasoning.

- ✖. **MAKING AN ARGUMENT** Let p be a positive constant, where the graph of $y = |x| + p$ is a vertical translation in the positive direction of the graph of $y = |x|$. Does this mean that the graph of $y = |x + p|$ is a horizontal translation in the positive direction of the graph of $y = |x|$? Explain.

- ✖. **DIG DEEPER** Write the vertex of the absolute value function $f(x) = |ax - h| + k$ in terms of a , h , and k .

REVIEW & REFRESH

59. Find the slope of the line.



60. Let $f(t)$ be the outside temperature (in degrees Celsius) t hours after 9 A.M. Explain the meaning of each statement.

- a. $f(4) = 30$ b. $f(m) = 28.9$
 c. $f(2) = f(9)$ d. $f(6.5) > f(0)$

In Exercises 61–64, solve the inequality. Graph the solution, if possible.

61. $2a - 7 \leq -2$
 62. $-3(2p + 4) > -6p - 5$
 63. $4(3h + 1.5) \geq 6(2h - 2)$
 64. $-4(x + 6) < 2(2x - 9)$

In Exercises 65 and 66, use the graphs of f and g to describe the transformation from the graph of f to the graph of g .

65. $f(x) = -\frac{1}{2}x$; $g(x) = f(x + 2)$
 66. $f(x) = 3x - 1$; $g(x) = -f(x)$

67. **MODELING REAL LIFE** You have \$15 to purchase pecans and walnuts. The equation $12x + 7.5y = 15$ models this situation, where x is the number of pounds of pecans and y is the number of pounds of walnuts.

- a. Interpret the terms and coefficients in the equation.
 b. Graph the equation. Interpret the intercepts.

In Exercises 68 and 69, solve the equation.

68. $-4|2x - 3| + 12 = -8$
 69. $|x + 4| = |5x + 2|$

70. **OPEN-ENDED** Draw a graph that does *not* represent a function.

- ✖. Convert 160 meters per minute to feet per second. Round your answer to the nearest hundredth.

72. **STRUCTURE** Compare the graphs. Find the values of h and k .

