2.5 Solving Compound Inequalities

GO DIGITAL

Learning Target Write and solve compound inequalities.

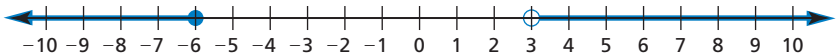
Success Criteria
- I can write word sentences as compound inequalities.
- I can solve compound inequalities.
- I can graph solutions of compound inequalities.

EXPLORE IT! Describing Intervals on the Real Number Line

Work with a partner. In parts (a)–(h), use two inequalities to describe the interval. Explain your reasoning.

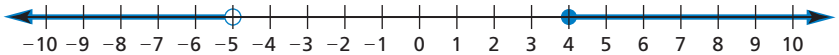
a. Half-Open Interval

b.



c. Half-Open Interval

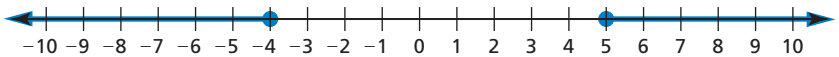
d.



e. Closed Interval

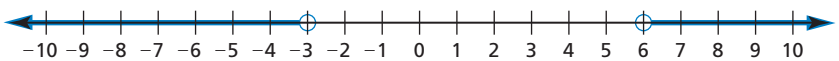
Communicate Precisely
Describe the difference between a closed interval and an open interval.

f.

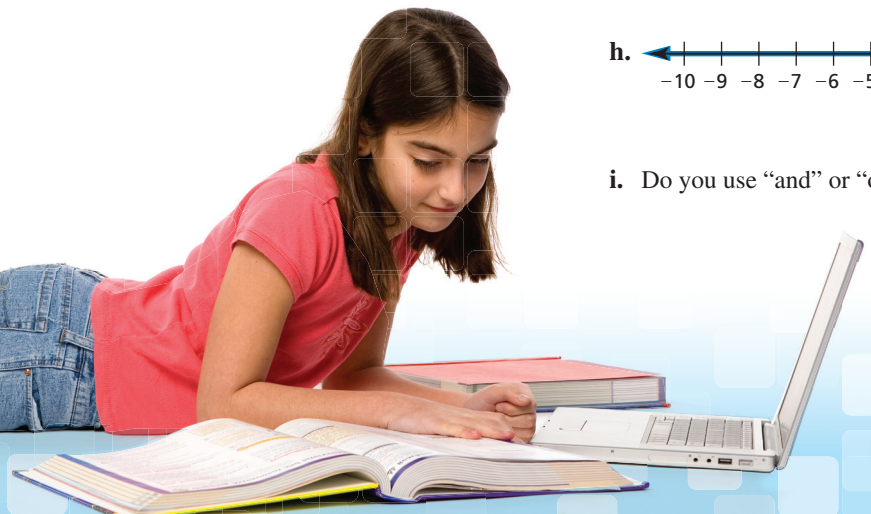


g. Open Interval

h.



i. Do you use "and" or "or" when writing the inequalities for each graph?



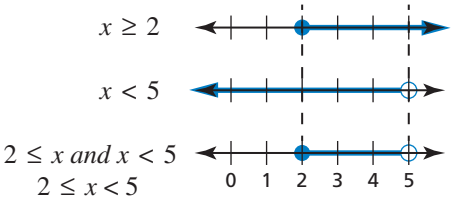
Writing and Graphing Compound Inequalities

A **compound inequality** is an inequality formed by joining two inequalities with the word "and" or the word "or."

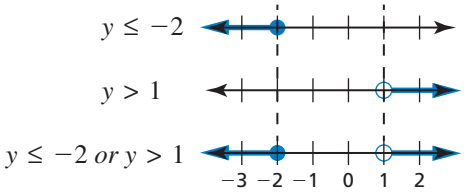
WORDS AND MATH

The word *compound* can have many meanings, such as a chemical mixture, a group of buildings, or a word made from more than one word. All of these meanings have something in common—they represent something that is made from more than one thing.

The graph of a compound inequality with "and" is the *intersection* of the graphs of the inequalities. The graph shows numbers that are solutions of *both* inequalities.



The graph of a compound inequality with "or" is the *union* of the graphs of the inequalities. The graph shows numbers that are solutions of *either* inequality.



Vocabulary

AZ VOCAB

compound inequality, *p. 102*

EXAMPLE 1 Writing and Graphing Compound Inequalities

WATCH

Write each sentence as an inequality. Graph each inequality.

a. A number x is greater than -8 and less than or equal to 4.

b. A number y is at most 0 or at least $\frac{3}{2}$.

SOLUTION

a. A number x is greater than -8 and less than or equal to 4.

$x > -8$ *and* $x \leq 4$

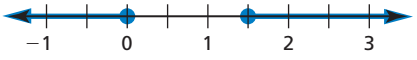
An inequality is $-8 < x \leq 4$.

Graph the intersection of the graphs of $x > -8$ and $x \leq 4$.

b. A number y is at most 0 or at least $\frac{3}{2}$.

$y \leq 0$ *or* $y \geq \frac{3}{2}$

An inequality is $y \leq 0$ *or* $y \geq \frac{3}{2}$.



Graph the union of the graphs of $y \leq 0$ and $y \geq \frac{3}{2}$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Write the sentence as an inequality. Graph the inequality.

1. A number d is more than 0 and less than 10.
2. A number a is fewer than -6 or no less than -3.
3. **WRITING** Compare the graph of $-6 \leq x \leq -4$ with the graph of $x \leq -6$ *or* $x \geq -4$.
4. **WHICH ONE DOESN'T BELONG?** Which compound inequality does *not* belong with the other three? Explain your reasoning.

$a > 4$ *or* $a < -3$ | $a < -2$ *or* $a > 8$ | $a > 7$ *or* $a < -5$ | $a < 6$ *or* $a > -9$

Solving Compound Inequalities

You can solve a compound inequality by solving two inequalities separately. When a compound inequality with "and" is written as a single inequality, you can solve the inequality by performing the same operation on each expression.

EXAMPLE 2 Solving Compound Inequalities with "And"

Solve each inequality. Graph each solution.

a. $-4 < x - 2 < 3$ **b.** $-3 < -2x + 1 \leq 9$

SOLUTION

a. Separate the compound inequality into two inequalities, then solve.

$-4 < x - 2$ *and* $x - 2 < 3$ Write two inequalities.

$+2 \quad +2$ $+2 \quad +2$ Addition Property of Inequality

$-2 < x$ *and* $x < 5$ Simplify.

▶ The solution is $-2 < x < 5$.

Look for Structure
In the inequality $-4 < x - 2 < 3$, what do you know about the quantity $x - 2$? How does this help you begin to solve?

b. $-3 < -2x + 1 \leq 9$ Write the inequality.

$-1 \quad -1 \quad -1$ Subtraction Property of Inequality

$-4 < -2x \leq 8$ Simplify.

$\frac{-4}{-2} > \frac{-2x}{-2} \geq \frac{8}{-2}$ Use the Division Property of Inequality. Reverse each inequality symbol.

$2 > x \geq -4$ Simplify.

▶ The solution is $-4 \leq x < 2$.

EXAMPLE 3 Solving a Compound Inequality with "Or"

Solve $3y - 5 < -8$ *or* $2y - 1 > 5$. Graph the solution.

SOLUTION

$3y - 5 < -8$ *or* $2y - 1 > 5$ Write the inequality.

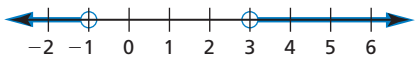
$+5 \quad +5$ $+1 \quad +1$ Addition Property of Inequality

$3y < -3$ $2y > 6$ Simplify.

$\frac{3y}{3} < \frac{-3}{3}$ $\frac{2y}{2} > \frac{6}{2}$ Division Property of Inequality

$y < -1$ *or* $y > 3$ Simplify.

▶ The solution is $y < -1$ *or* $y > 3$.



SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Solve the inequality. Graph the solution.

5. $5 \leq m + 4 < 10$

6. $-3 < \frac{2}{3}k - 5 < 0$

7. $4c + 3 \leq -5$ *or* $c - 8 > -1$

8. $2p + 1 < -4$ *or* $3 - 8p \leq -1$

9. OPEN-ENDED Write a compound inequality that has a solution of all real numbers except $x = 0$.

Operating temperature:
0°C to 35°C

Solving Real-Life Problems

EXAMPLE 4 **Modeling Real Life** WATCH INFO

An electronic device may fail outside of its operating temperature range. Write an inequality that represents the possible operating temperatures (in degrees Fahrenheit) of the smartphone. Then describe a situation in which the phone may be outside of the operating range.

SOLUTION

1. **Understand the Problem** You know the operating temperature range in degrees Celsius. You are asked to represent the range in degrees Fahrenheit and to describe a situation outside of this range.

2. **Make a Plan** Write a compound inequality in degrees Celsius C. Use the formula $C = \frac{5}{9}(F - 32)$ to rewrite the inequality in degrees Fahrenheit F. Then solve the inequality and describe a situation outside of this range.

3. **Solve and Check**

$$0 \le C \le 35$$ Write the inequality using C.

$$0 \le \frac{5}{9}(F - 32) \le 35$$ Substitute $\frac{5}{9}(F - 32)$ for C.

$$\frac{9}{5} \cdot 0 \le \frac{9}{5} \cdot \frac{5}{9}(F - 32) \le \frac{9}{5} \cdot 35$$ Multiplication Property of Inequality

$$0 \le F - 32 \le 63$$ Simplify.

$$+ 32 \quad + 32 \quad + 32$$ Addition Property of Inequality

$$32 \le F \le 95$$ Simplify.

▶ A solution is $32 \le F \le 95$. So, the operating temperature range of the smartphone is 32°F to 95°F. Someone might leave the phone in a car on a hot day, where temperatures can exceed 150°F.

Check

You can use the formula $C = \frac{5}{9}(F - 32)$ to check that your answer is correct. Substitute 32 and 95 for F in the formula to verify that 0°C and 35°C are the minimum and maximum operating temperatures in degrees Celsius. ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

10. A pair of winter boots are rated for temperatures from −40°C to 15°C. Write an inequality that represents the temperature rating (in degrees Fahrenheit) of the boots.

11. Birdwatchers record the types of birds they see or hear. The graph shows results from a location in Canada. Write an inequality that represents the range in the percents of birdwatchers who saw or heard a Black-and-white Warbler from July 1 to September 15.